PORE DIAMETERS IN POROUS METAL FROM WOVEN STRIPS

A. V. Kurpatenkov, V. M. Polyaev,
and A. L. Sintsov

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Theoretical formulas are described for calculating the pore diameters in undeformed woven strips and in metals based on them.

Porous metals formed from woven strips are mainly used as filters. It has been shown by experiment that such porous metals can also be used to accelerate heat transfer in compact heat exchangers.

It is necessary to know a certain internal scale for the medium in order to calculate the flow and heat transfer in the pores. While a granular layer or a bed of spheres can be characterized by the mean hydraulic diameter, which is proportional to the ratio of the porosity to the specific surface [2], the scale most widely used for a porous metal is the mean pore diameter. The latter is determined from the pressure difference on blowing a gas through the wetted specimen [3].

Measurements have been reported [3, 4] on the dependence of pore diameter on porosity for metals of various types. While experimental studies present valuable factual evidence, they have certain disadvantages, since in particular they do not enable one to establish the effects of the geometrical characteristics of the initial strip on the metal pore diameter, so one cannot forecast the value, while another important point in our view is that one cannot establish how completely this quantity characterizes the geometrical structure of the pore space.

Here we present an analytical approach to determining the pore diameter, while enables one to eliminate the above disadvantages in experimental studies.

The initial point in determining the metal pore diameter is a geometrical model for an undeformed strip, where the pore diameter can be calculated theoretically. The latter is of independent interest in connection with separating gas–liquid mixtures under conditions of low gravity [5].

In turn, the pore diameter calculation is based on examining the variation in hydraulic diameter over the thickness and finding the value least in a certain sense. It has been shown [6] that it is correct to identify the pore diameter with the hydraulic diameter for a fibrous material.

Here we consider dense strips (without cells transmitting light) made by weaving. The strips are characterized by the following initial parameters: warp diameter d_o, weft diameter d_u, number of wires in warp n_o, and number of wefts n_u per unit length. The type of weaving is defined by the quantity $\varkappa$: $\varkappa=0$ linen, 0.5 semiserge, and 1 serge. Pictures of the different types of weaving can be found in [7]. The dimensional parameters are accompanied by dimensionless ones:

$$\nu_{u(o)} = n_{u(o)} d_u^0, \quad \delta_{u(o)} = d_{u(o)}/d_u^0, \quad \xi = (\delta_o + \delta_u)\nu_o,$$

where d_u^o is the standard diameter of a weft wire, which is taken as the characteristic scale. The difference of d_u^o from d_u is explained below.

We now give the expression for the strip porosity. The ratio of the volume of material to the area of a rectangle cut from it is readily shown to be equal to the sum $(n_u d_u^2\theta + n_o d_o^2)\pi/4$, which on dividing by d_u^o to give a dimensional quantity gives $\Sigma_s = (\nu_u\delta_u^2\theta + \nu_o\delta_o^2)\pi/4$. Here θ is ratio of the length of a sufficiently large part of the axial line of the weft to the projection on the median plane. Simple geometrical considerations give a formula for θ:

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$$\Phi = \arcsin \xi,\ \omega = \varkappa n_o/n_u,\ \tau = \xi\Phi + \sqrt{1-\xi^2+\omega^2},\ t = \xi^2 + \sqrt{(1-\xi^2)^2+\omega^2},$$
$$\theta = (\tau + \varkappa t)/(1+\varkappa).$$

It should be noted that the wefts in serge and semiserge weaving have a certain inclination in the plane of the strip. The ratio of the inclination of the weft in a step between adjacent warps to the size of the step is equal to ω.

We have the dimensionless thickness and the porosity as

$$h = \delta_o + 2\delta_u,\ v_p = 1 - \Sigma_s/h. \tag{1}$$

Measurements show that the initial parameters often deviate appreciably from the standard ones, so they should be determined by direct measurement. However, this is not always possible. If the values of d_o, n_o, and n_u are unknown, one has to use their standard values. The standard value d_u^o is usually larger than the real d_u because of the stretching in making the strip. Allowance can be made for this in the following approximate fashion. We assume that an initially straight wire in the weft of diameter d_u^o is stretched by a factor θ during manufacture. Then as the volume is constant, the diameter of the wire d_u will take the value $d_u\sqrt{\theta} = d_u^o$. The quantity θ is dependent on d_u via ξ, so the solution to the equation can be obtained only numerically, for example by successive approximation, with d_u^o taken as the initial value. Calculations show that the value of d_u can be determined quite accurately after two or three iterations.

We introduce a coordinate system. We bring the xOy plane into coincidence with the median plane of the strip. The x axis is directed along the warp, while the z axis is vertically upwards. From the strip we dissect out a rectangle with sides L_o and L_u, which are directed along the warp and weft, respectively. We denote by $S_u(S_o)$ the area of cross section of one wire in the weft (warp) in a plane parallel to xOy. Then the ratio of the free area to the area of the rectangle (working section) is given by $\psi = 1 - \varsigma,\ \varsigma = n_u S_u/L_u + n_o S_o/L_o$.

We denote by l_u the pitch between the warps. After passing through a certain number of warps, the weft of wire returns to the initial position. We denote the length of this path by T_u. It is clear that T_u/l_u is an integer dependent on the type of weaving. That is, for linen weaving it is 2, while for semiserge it is 3, and for serge it is 4, or in short it is $2\,(1+\varkappa)$.

We give the name $\bar{n}$ type to wefts running parallel to the median plane between adjacent warps. The number of these per unit length is denoted by $\bar{n}_u$, while the area of the z section in the step between adjacent warps is $\bar{S}_u$. The wefts intersecting the median plane are said to be of $\tilde{n}$ type, and we introduce the corresponding symbols $\tilde{n}_u$, $\tilde{S}_u$. We note that the following equations apply:

$$\bar{n}_u = \varkappa n_u/(1+\varkappa),\ \ \tilde{n}_u = n_u/(1+\varkappa). \tag{2}$$

Without loss of generality, L_u can be taken as equal to T_u. Then we express T_u in terms of l_u and $\varkappa$ to get for S_u/L_u

$$S_u/L_u = (\bar{S}_u + 2\tilde{S}_u)/\{2\,(1+\varkappa)\,l_u\}. \tag{3}$$

We note that for linen weaving $\bar{S}_u$ must be taken as zero, while $\bar{S}_u$ for semiserge should be considered only on one side of the strip, with $\bar{S}_u = 0$ on the other.

The warps in a strip of dense type are rectilinear and lie in the median plane, so the expression for S_o/L_o takes a particularly simple form.

We denote by P_u and P_o the perimeters of the sections for wires in the weft and warp in the rectangle. If we substitute P_u and P_o instead of S_u and S_o into the formula for ζ, then the resulting quantity p will be equal to the ratio of the perimeters of the section of all the wires to the area of the rectangle. A difference from ζ is that p is dimensional. Instead of p we use the dimensionless quantity $\pi = pd_u^o$. Then

$$\pi = \nu_u P_u/L_u + \nu_o P_o/L_o. \tag{4}$$

Then (3) with $\bar{S}_u$ replaced by $\bar{P}_u$ and $\tilde{S}_u$ by $\tilde{P}_u$ enables one to calculate P_u/L_u. Then a remark made about $\bar{S}_u$ applies also to $\bar{P}_u$.

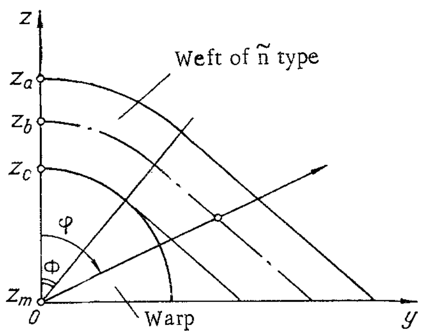

Fig. 1. Characteristic z coordinates for the $\delta_h(z)$ dependence.

Clearly, the ratio $4\psi/\pi$ is equal to the dimensionless hydraulic diameter of a certain z section. We denote this by δ_h.

We now note certain features of ψ, π, and δ_h as functions of z. Figure 1 gives the coordinates used below for sections in the strip. It is readily seen that $\bar{S}_u$, $\tilde{S}_u$, S_o are continuous in z. Therefore, ς and ψ are also continuous. The situation is different with π. In fact, the values of $\bar{P}_u$ and P_o are independent of z, which means that $\lim \pi$ for $z \to z_c + 0$ is not equal to $\lim \pi$ for $z \to z_c - 0$, and therefore π has a discontinuity at z_c, and consequently so does δ_h. Also, ψ is equal to one at the point z_a for any type of weaving. The value of π, on the other hand, at this point is zero for the linen-weave strip but different from zero for serge. Consequently, δ_h increases without limit for $z \to z_a$ for linen-type strip and has a finite limit for serge.

We now calculate $\tilde{S}_u$, $\tilde{P}_u$. Figure 1 shows that the surface of the weft of $\tilde{n}$ type consists of parts of a torus and cylinder; the z sections of the surface are ellipses for $\varphi \geqslant \Phi$, while for $\varphi < \Phi$ they are curves of fourth order (sections of a torus):

$$x^2 = 1 - (\sqrt{y^2 + z^2} - z_b)^2. \tag{5}$$

Note that we have used $0.5d_u^0$ as the scale in (5). Sections of a torus are very close in features to Cassini ovals. However, it can be shown that for a given torus the shapes of only two of the z sections are identical with Cassini ovals. In the general case, neither the area nor the perimeter of the curve of (5) (quantities equal to $\tilde{S}_u$, $\tilde{P}_u$ apart from scales) is expressed by known functions. One can determine $\tilde{S}_u$, $\tilde{P}_u$ only by numerical integration.

We transform (1) and (4) using (2) and (3):

$$\varsigma = \nu_o \{\tilde{\nu}_u (0{,}5\, \bar{S}_u' + \tilde{S}_u') + S_o/(d_u^0 L_o)\}, \tag{6}$$

$$\pi = \nu_o \{\tilde{\nu}_u (0{,}5\, \bar{P}_u' + \tilde{P}_u') + P_o/L_o\}. \tag{7}$$

Here S' is referred to $(d_u^0)^2$ and P' to d_u^0. The determination of $\bar{S}_u$, $\bar{P}_u$, S_o, P_o clearly does not cause difficulty, since one uses the corresponding characteristics of cylinder sections. One derives $\tilde{S}_u$ and $\tilde{P}_u$ from numerical integration, and from (6) and (7) one can then calculate ζ and π, and thus can determine δ_h for a certain z section.

Figure 2 shows the results from such calculations for a series of serge strips. Clearly, the $\delta_h(z)$ dependence for serge and linen strips is symmetrical with respect to the median plane. Correspondingly, Fig. 2 shows that $\delta_h(z)$ for serge strips has three characteristic features: near the points z_b, $-z_b$, and at z_m. The $\delta_h(z)$ dependence for linen weave is not shown in Fig. 2, but over the part $(-z_c;\ z_c)$ it hardly differs from the relationship for the corresponding serge: a small difference is introduced by the inclination of the weft in the latter. Over the part $(z_c;\ z_a)$, $(-z_a;\ -z_c)$, δ_h tends monotonically to infinity. Therefore, the $\delta_h(z)$ curve for linen-type strips has the form of two funnels with one singular point symmetrical with respect to the median plane. The $\delta_h(z)$ dependence for semiserge, neglecting the slight effect of the weft inclination, coincides with that for serge on one side of the median plane and with that for linen weave on the other. These are the general features of the structure in the equivalent hydraulic channel for dense weaves of these three types.

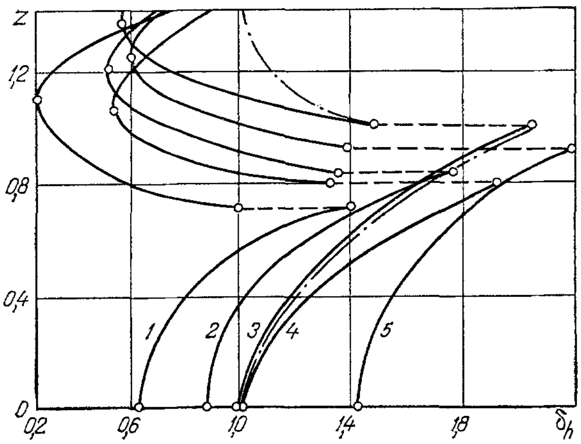

Fig. 2. Behavior of hydraulic diameter and pore diameter for certain dense strips with serge weaving: solid lines: calculated $\delta_h(z)$; points from top downwards: $\delta_{h,\ ac}$, $\delta_{h,\ +}$, $\delta_{h,\ -}$, $\delta_{h,\ m}$; dot-dash line: approximate coarse of $\delta_p(z)$ for S685 strip; 1) S200; 2) S450; 3) S685; 4) S120; 5) S80.

We now give formulas for calculating $\delta_h(z_m)$, which is used below. The trace of the $\tilde{n}$-type weft in the median plane is an ellipse, whose major and minor semiaxes are equal in the absence of inclination (Fig. 1): $b_o = 0.5\delta_u/\xi$, $a = 0.5\delta_u$. The inclination of the wefts at angle β leads to a certain increase in the major semiaxis: $b_\beta = b_0\sqrt{1+\omega^2/(1-\xi^2)}$.

We determine the perimeter and area of the ellipse $\tilde{P}_u'$, $\tilde{S}_u'$, and then for the quantity $\delta_{h,m}$

$$\delta_{h,m} = 4\,(1 - \nu_o\,\{\tilde{\nu}_u \tilde{S}_u' + \delta_o\})/(\nu_o\{\tilde{\nu}_u \tilde{P}_u' + 2\}).$$

The pore diameter in a porous material is defined by the following formula [3] in traditional symbols:

$$\delta_p = 4\,\sigma\cos\Theta/(\Delta P d_u^0). \tag{8}$$

The various z coordinates in the interface between the phases in the strip correspond to different values of the pressure difference and on account of (8) to different values of the pore diameter. Therefore, we have a function $\delta_p(z)$ analogous to $\delta_h(z)$. By pore diameter we mean the minimum in δ_p with respect to z.

We have seen above that the δ_p determined from (8) for a fibrous material is virtually equal to the hydraulic diameter δ_h. In application to undeformed strips, this enables one to identify the least value of the hydraulic diameter with the pore diameter. In particular, for linen-weave strips we get as follows on the basis of $\delta_h(z)$:

$$\delta_p = \delta_h(z_m) = \delta_{h,m},\ \varkappa = 0. \tag{9}$$

At first sight, one determines the pore diameter for serge weave by seeking the minimum in δ_h with respect to z. However, this is not so, and the reason lies in the specific form of the z sections of a serge strip near the point z_b. It is readily seen in fact that the working section at $z \approx z_b$ is a curvilinear quadrilateral. The shape near the vertices resembles sharp-ended whiskers with a large perimeter and relatively small area. This means that the phase interface in the region of point z_b bears not on the entire perimeter but only on a certain active part not extending into the regions adjoining the vertices, and as it were it smooths the sharp-ended form. It is clearly impossible to determine the actual area and perimeter of the phase interface near z_b and thus the pore diameter.

Figure 2 shows the qualitative dependence of δ_p on z for S685 strip.

However, the function $\delta_p(z)$ can be identified with $\delta_h(z)$ in the part $(z_m;\ z_c)$. The minimum in δ_h in this part evidently occurs for $z = z_m$: $\delta_{p,\ m} = \delta_{h,\ m}$. The minimum value of δ_p in the part $(z_c;\ z_a)$, namely $\delta_{p,\ ac}$, is not known, as noted. We denote the limit to δ_h

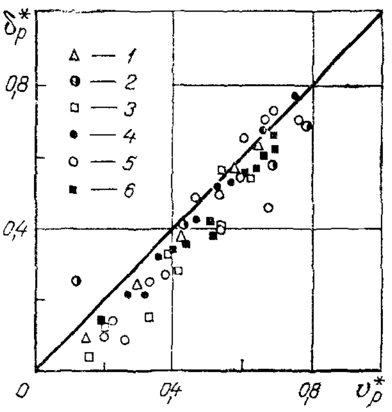

Fig. 3. Experimental data on pore diameter and current porosity for a series of metals: 1) P80, points calculated from the empirical correlation (29) of [3]: 2) S80 [13]; 3) S120 [14]; 4) S200 [14]; 5) S450 [14]; 6) S685 [14].

TABLE 1. Comparison of Calculated Pore Diameters with Measured Ones

Strip	d_o, μm	d_u, μm	Calculated d_p, μm			Observed d_p, μm	References
			minimal	standard	maximal		
325/2300	38	25	8,5	11,9	16,6	10,0	[5]
						10,0—13,0	[8]
						12,6—13,9	[9]
						13,3	[10]
						14,5	[12]
250/1400	55	40	12,9	13,7	18,1	13,5	[8]
200/1400	70	40	13,4	17,3	25,3	13,4	[11]
						15,0	[5]
						15,0—17,0	[8]
						16,8	[10]
						21,1	[9]
						21,4	[12]
165/1400	70	40	18,5	26,1	34,8	18,6	[11]
						21,0—22,0	[8]
165/800	70	50	18,4	24,1	32,3	22,7	[11]
850/155	100	30	13,8	24,6	36,6	23,0	[8]
720/1						31,2	[12]
720/150	110	35	8,6	22,2	33,6	32,9	[10]
670/120	120	32	23,7	36,8	51,5	31,0	[8]

for $z \to z_c + 0$ as $\delta_{h,+}$. Then clearly $\delta_{p,ac}$ lies between $\delta_{h,+}$ and $\delta_{h,ac}$, the minimum in δ_h in this part. The approximation of $\delta_{p,x}$ to $\delta_{p,ac}$ is recommended as being determined as the arithmetic mean between $\delta_{h,+}$ and $\delta_{h,ac}$. Then we naturally have the following approximate formula for the pore diameter in a serge strip:

$$\delta_p = \min(\delta_{p,m};\ \delta_{p,x}). \tag{10}$$

Table 1 compared (9) and (10) with fairly extensive experimental evidence. By specifying permissible standard deviations (about ±10%) for the initial parameters, we determine the minimum and maximum possible values of δ_p, which are also given in the table. Table 1 indicates that (9) and (10) can be used to calculate pore diameters in undeformed strips.

Before we consider the dependence of δ_p on the deformation of a packet of strips, or in other words on the porosity of the resulting metal, it is desirable to elucidate the number of quantities that govern the value of δ_p. The answer is a trivial consequence of the π theorem: d_p is determined by the four initial strip parameters, so d_p/d_u will be determined by three parameters, for example $n_o d_u$, $n_u d_u$, d_o/d_u. If we neglect the difference between

TABLE 2. Calculated Characteristics of Certain Dense Strips

Strip	$\varkappa$	GOST or TU	Standard weft diameter d_u^0, μm	Pore diameter			Porosity v_p^0	$\frac{\delta_p^0}{[v_p^0]^{1,25}}$	ξ	δ_u	δ_o
				$\delta_{p,m}$	$\delta_{p,x}$	δ_p^0					
P80	0	GOST 3187—76	180	1,51	—	1,51	0,679	2,44	0,363	0,967	1,56
S80	1	GOST 3187—76	200	1,43	1,01	1,01	0,451	2,75	0,436	0,975	1,75
S120	1	GOST 3187—76	160	1,18	0,945	0,945	0,429	2,73	0,486	0,968	1,56
S200	1	GOST 3187—76	140	0,632	0,689	0,632	0,343	2,41	0,664	0,942	1,43
S450	1	TU 14—4—697—76	55	0,875	0,945	0,875	0,407	2,69	0,639	0,946	1,64
S685	1	TU 14—4—697—76	32	0,999	1,04	0,999	0,419	2,96	0,646	0,946	1,99

d_u and d_u^o, which is unimportant here, we get that δ_p is a function of three arguments: ν_o, ν_u, δ_o. We use $\varkappa$ and ξ together with ν_o and ν_u. This is clearly permissible.

Then it is clear that the pore diameter in a consolidated packet of strips δ_p^Δ is dependent not only on $\varkappa$, ξ, and δ_o but also on the degree of consolidation or the current porosity v_p^Δ:

$$\delta_p^\Delta = f(\varkappa,\ \xi,\ \delta_o,\ v_p^\Delta). \tag{11}$$

We denote the pore diameter and porosity in an undeformed strip by δ_p^o and v_p^o correspondingly. For a given   we can express ξ and δ_o in terms of δ_p^o and v_p^o, so we can transform (11):

$$\delta_p^\Delta = f(\varkappa,\ \delta_p^0,\ v_p^0,\ v_p^\Delta). \tag{12}$$

We introduce the symbols $\delta_p^* = \delta_p^\Delta/\delta_p^o$, $v_p^* = v_p^\Delta/v_p^o$.

Figure 3 gives experimental data for a series of metals in δ_p^*, v_p^* coordinates. Table 2 gives the characteristics of the initial strips calculated from the above formulas. From Fig. 3 we can rewrite (12) in a completely unexpected form:

$$\delta_p^* = f(v_p^*) \approx v_p^*. \tag{13}$$

Finally, for (13) one can recommend a more accurate approximation:

$$\delta_p^* = v_p^{*1,25}. \tag{14}$$

Formulas (1), (9), (10), and (14) constitute the main content of this study. They enable one to calculate pore diameters for undeformed strips and for porous metals based on them.

We can now determine how far the d_p^Δ and v_p^Δ characterize the geometry of the pore space in a given metal, or in other words whether one can have metals made of different strips of identical pore diameters with identical porosity. The answer is given from (14): for this purpose it is sufficient for the ratio $d_p^o/(v_p^o)^{1.25}$ to be the same in the two cases. Calculations show for example that in P80 and S120 strips the ratios differ by less than 1%. Therefore, porous metals made of P80 and S120 have identical pore diameters and identical porosity. On the other hand, the pore structures in these metals are completely different, because of the differences in geometrical structure in the initial strips: linen weave and serge.

We thus draw the important conclusion that d_p^Δ and v_p^Δ are inadequate to characterize these metals, and the implications are deeper than coincidence between the pore diameters.

The results show that any geometrical characteristic of such a metal is a function of five parameters such as the following: d_u^0, $\varkappa$, ξ, δ_0, v_p^Δ. This number is too large, and this raises the far from trivial question of reducing it. An example of the reduction is the use of the two quantities d_p^Δ, v_p^Δ. In accordance with (14), d_p^Δ is determined by the product of d_u^o by the corresponding power of v_p^Δ and the ratio $\delta_p^o/(v_p^o)^{1.25}$. Values from the latter are given in Table 2, and it is evident that the quantity is almost constant. Therefore, d_p^Δ is virtually a function of d_u^o and v_p^Δ, and use of v_p^Δ, d_p^Δ is almost equivalent to using v_p^Δ, d_u^o. However, it is completely clear that the porosity and the linear scale cannot completely characterize the structure of the pore space.

These results illustrate why there are no physical regularities in the hydraulic resistance laws derived using d_p^Δ, v_p^Δ for porous strip metals [4].

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